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LETTER TO THE EDITOR

# Analytic regularisation and Ward identity in a broken supersymmetric model

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**Abstract.** A Ward identity in a broken supersymmetric model involving a supersymmetry current is studied using analytic regularisation. We show, in the one-loop approximation, that this Ward identity is satisfied if the amplitudes are regularised by the analytic regularisation method.

Some time ago we analysed the possibility of using analytic regularisation in supersymmetric theories. In this context, we first studied the anomaly (Kumar and Fujii 1982) of the supercurrent in the supersymmetric Yang-Mills multiplet. Later, we used this regularisation scheme to study the Ward identities (Kumar and Fujii 1983) in the massless Wess-Zumino model.

Earlier, several authors studied the supersymmetric Ward identities in various models using different regularisation methods. Townsend and van Nieuwenhuizen (1979) have studied the Ward identities in the massless Wess-Zumino model (1974a, b) using dimensional regularisation ('t Hooft and Veltman 1972). Sezgin (1980) and Hagiwara and Majumdar (1981) have extended these results to the study of the Ward identities in the massive Wess-Zumino model. Capper *et al* (1980) studied the Ward identities in the vector supersymmetric model by using the dimensional reduction method (Siegel 1979).

In this letter we study a Ward identity in the Wess-Zumino model (Wess and Zumino 1974a, b) in the presence of symmetry breaking and mass terms. As is well known, in this model the spontaneous breaking of supersymmetry is unstable. We resort to explicit breaking of supersymmetry (Illiopoulos and Zumino 1974). The specific Ward identity we propose to study involves one supersymmetry current and a fermion field. The amplitudes entering in this identity are regularised by an analytic regularisation method (Bollini *et al* 1964, Speer 1968, 1974).

Very recently a similar regularisation scheme (Alfaro 1985) has been proposed in the context of stochastic quantisation of field theories (Parisi and Wu 1981, Breit *et al* 1984). Alfaro (1985) also made an attempt to establish a direct contact between stochastic regularisation and analytic regularisation.

The present work differs from our earlier one (Kumar and Fujii 1983) in several ways. First of all we consider the Wess-Zumino model in the presence of mass terms. Secondly, the supersymmetry is broken by an explicit term in the Lagrangian. Thirdly, the Ward identity we consider here is different in the sense that it involves one supersymmetry current whereas in our earlier work we considered Ward identities involving only various fields. The model we consider is described by the following

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Lagrangian (Wess and Zumino 1974a, b, Illiopoulos and Zumino 1974):

$$L=-\tfrac{1}{2}[(\partial_\mu A)^2+(\partial_\mu B)^2+\bar{\psi}\not{\partial}\psi-F^2-G^2]+m(FA+GB-\tfrac{1}{2}\bar{\psi}\psi)$$
$$+g[FA^2-FB^2+2GAB-\bar{\psi}(A-\mathrm{i}\gamma_5 B)\psi)]-cA. \tag{1}$$

The fields in this model are two scalar fields $A$ and $F$, two pseudoscalar fields $B$ and $G$ and a Majorana spinor $\psi$. Under infinitesimal supersymmetry transformations these fields transform as†

$$\delta A=\bar{\varepsilon}\psi \tag{2a}$$

$$\delta B=\mathrm{i}\bar{\varepsilon}\gamma_5\psi \tag{2b}$$

$$\delta F=\bar{\varepsilon}\not{\partial}\psi \tag{2c}$$

$$\delta G=\mathrm{i}\bar{\varepsilon}\gamma_5\not{\partial}\psi \tag{2d}$$

$$\delta\psi=\not{\partial}(A+\mathrm{i}\gamma_5 B)\varepsilon+(F+\mathrm{i}\gamma_5 G)\varepsilon \tag{2e}$$

where $\varepsilon$ is a constant anticommuting Majorana spinor.

Apart from the last term, the Lagrangian in equation (1) changes under the supersymmetry transformations of equations (2) by a total derivative. In the symmetric case ($c=0$) the Lagrangian is invariant under the transformation of equations (2) and one can construct the conserved Noether current

$$J_\mu=[\not{\partial}(A-\mathrm{i}\gamma_5 B)+m(A+\mathrm{i}\gamma_5 B)+(A+\mathrm{i}\gamma_5 B)^2]\gamma_\mu\psi. \tag{3}$$

The last term, $-cA$ ($c$ is a constant), in equation (1) breaks the supersymmetry and the Noether current in equation (3) is no longer conserved; rather, one has

$$\partial_\mu J_\mu=c\psi. \tag{4}$$

We can eliminate the linear term, $-cA$, from equation (1) by shifting the fields $A$ and $F$, $A\to A+a$, $F\to F+f$ and demanding that the fields $A$ and $F$ have vanishing vacuum expectation values to all orders. This gives us two consistency conditions which determine the parameters $a$ and $f$. In the tree approximation these conditions are

$$f+ma+ga^2=0 \tag{5a}$$

$$mf+2gaf-c=0. \tag{5b}$$

Further, the masses of the fields $\psi$, $A$ and $B$ are no longer equal but are given in the lowest order by the following relations:

$$m_\psi=m+2ga \tag{6a}$$

$$m_A^2=m_\psi^2-2gf \tag{6b}$$

$$m_B^2=m_\psi^2+2gf. \tag{6c}$$

Now with the shift of the fields the current in equation (3) also changes and it takes the form

$$J_\mu=[\not{\partial}(A-\mathrm{i}\gamma_5 B)+m_\psi(A+\mathrm{i}\gamma_5 B)+g(A+\mathrm{i}\gamma_5 B)^2+(m_\psi a-ga^2)]\gamma_\mu\psi. \tag{7}$$

† We use the following convention for the $\gamma$ matrices: the four $\gamma$ matrices are Hermitian and satisfy $\{\gamma_\mu,\gamma_\nu\}=2\delta_{\mu\nu}$ ($\mu,\nu=1,2,3,4$). $\bar{\psi}$ is defined by $\psi^+\gamma_4$ and $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$.

The Ward identity we intend to study in this letter is the following (de Wit 1975)

$$\partial_\mu\langle T(J_\mu(x)\bar{\psi}(0))\rangle_0 = c\langle T(\psi(x)\bar{\psi}(0))\rangle_0 + \delta^4(x)f. \tag{8}$$

One can write this Ward identity in momentum space as

$$\mathrm{i}p_\mu\Gamma^\psi_\mu(p) = c - \mathrm{i}fS_\psi^{-1}(p). \tag{9}$$

In equation (9) $\Gamma^\psi_\mu(p)$ is the irreducible Green function involving a supersymmetry current and a fermion. $S_\psi^{-1}$ is the inverse fermion propagator.

This identity has been studied in detail by de Wit (1975) using the higher-order derivative regularisation. As he points out, this Ward identity is important in proving the existence of zero-mass Goldstone fermions when spontaneously broken realisation of supersymmetry is considered.

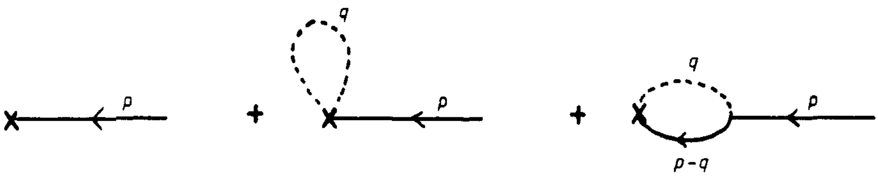


**Figure 1.** One-loop contribution to $\Gamma^\psi_\mu$. Full lines represent the fermions and broken lines represent the bosons. The crosses represent $J_\mu$.

We shall now calculate $\Gamma^\psi_\mu(p)$ and $S_\psi^{-1}(p)$ on the one-loop approximation, using analytic regularisation and verify the Ward identity in equation (9). The diagrams contributing to $\Gamma^\psi_\mu$ to this order are shown in figure 1. The amplitude corresponding to figure 1 is given by

$$\begin{aligned}\Gamma^\psi_\mu(p) = (m_\psi a - ga^2)\gamma_\mu - \frac{\mathrm{i}g}{(2\pi)^4}\int \mathrm{d}^4q[D_{AA}(q) - D_{BB}(q)]\gamma_\mu \\ + \frac{2g}{(2\pi)^4}\int \mathrm{d}^4q\,\mathrm{i}\not{q}\gamma_\mu[D_{AA}(q)S_\psi(p-q) + D_{BB}(q)\gamma_5S_\psi(p-q)\gamma_5] \\ + \frac{2g}{(2\pi)^4}\int \mathrm{d}^4q\,m_\psi\gamma_\mu[D_{AA}(q)S_\psi(p-q) - D_{BB}(q)\gamma_5S_\psi(p-q)\gamma_5].\end{aligned} \tag{10}$$

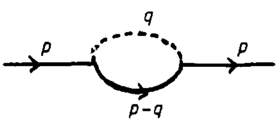


**Figure 2.** Self-energy contribution to fermion propagator $S_\psi$.

Similarly, the one-loop contribution to the fermion propagator is represented in figure 2. Taking this self-energy contribution into account we can write $S_\psi^{-1}(p)$ as

$$S_\psi^{-1}(p) = \not{p} - \mathrm{i}m_\psi - \frac{4\mathrm{i}g^2}{(2\pi)^4}\int \mathrm{d}^4q[D_{AA}(q)S_\psi(p-q) - D_{BB}(q)\gamma_5S_\psi(p-q)\gamma_5]. \tag{11}$$

Figure 3 represents the two consistency conditions in the one-loop order which determine the parameters $a$ and $f$. Mathematically these conditions to this order are

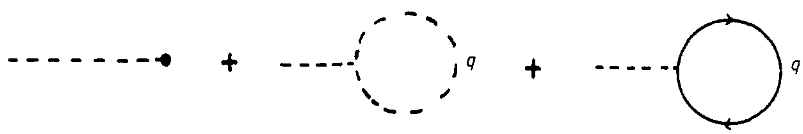

**Figure 3.** Diagrammatic representation of the consistency conditions that determine the parameters $a$ and $f$.

given by

$$c - m_\psi f + \frac{\mathrm{i}g}{(2\pi)^4}\int \mathrm{d}^4q[2D_{AF}(q) + 2D_{BG}(q) - \mathrm{i}\,\mathrm{Tr}(S_\psi(q))] = 0 \tag{12a}$$

$$m_\psi a - ga^2 + f - \frac{\mathrm{i}g}{(2\pi)^4}\int \mathrm{d}^4q[D_{AA}(q) - D_{BB}(q)] = 0. \tag{12b}$$

The various zeroth-order propagators entering in equations (10)-(12) are given below:

$$D_{AA}(q) = \frac{1}{q^2 + m_A^2} \tag{13a}$$

$$D_{BB}(q) = \frac{1}{q^2 + m_B^2} \tag{13b}$$

$$S_\psi(q) = \frac{1}{\not{q} - \mathrm{i}m_\psi} \tag{13c}$$

$$D_{AF} = -m_\psi D_{AA} \tag{13d}$$

$$D_{BG} = -m_\psi D_{BB}. \tag{13e}$$

We now substitute the values of $\Gamma_\mu^\psi$ and $S_\psi^{-1}$ from equations (10) and (11) into the Ward identity, equation (9). After using the consistency condition of equation (12$b$) and rearranging various terms the Ward identity in equation (9) can be written as

$$\begin{aligned}\int \mathrm{d}^4q\,\not{q}\not{p}[D_{AA}(q)S_\psi(p-q) + D_{BB}(q)\gamma_5 S_\psi(p-q)\gamma_5]\\ = -\frac{8\pi^4}{g}(c - m_\psi f) + (2gf + \mathrm{i}m_\psi\not{p})\\ \times\int \mathrm{d}^4q[D_{AA}(q)S_\psi(p-q) - D_{BB}(q)\gamma_5 S_\psi(p-q)\gamma_5].\end{aligned} \tag{14}$$

One can see that the integrals on the left-hand side (LHS) of this equation are quadratically divergent and on the right-hand side (RHS) linearly divergent. In order to evaluate these integrals unambiguously one uses a regularisation scheme. This is where we introduce analytic regularisation (Bollini *et al* 1964, Speer 1968, 1974). As is known, in this regularisation scheme one modifies the propagators by introducing an exponent ($\lambda$) in the denominators which regularises the divergences. All the divergences then appear as poles in this regularising exponent. Finally, one employs a minimal subtraction scheme to remove these divergences and takes the limit $\lambda \to 0$. In our case we adopt the following strategy. To evaluate the divergent integrals on the LHS of equation (14) we modify only the fermion propagator whereas on the RHS we modify only the boson propagators, i.e.

$$S_\psi(q) \to \frac{(\not{q} + \mathrm{i}m_\psi)}{(q^2 + m_\psi^2)^{1+\lambda}} \tag{15a}$$

$$D_{AA}(q) \to \frac{1}{(q^2 + m_A^2)^{1+\lambda}} \tag{15b}$$

$$D_{BB}(q) \to \frac{1}{(q^2 + m_B^2)^{1+\lambda}}. \tag{15c}$$

This point is very important in the verification of the Ward identity. If we modify only one type of propagator then the Ward identity is violated by finite terms. In our earlier work (Kumar and Fujii 1982, 1983) an almost similar procedure was adopted. This we feel is typical of supersymmetric theories where cancellation between the Bose and Fermi contributions in any given calculation takes place.

In the above expressions (15*a*)-(15*c*) the exponent $\lambda$ is the regularising parameter. With these modifications we calculate the momentum integrals in equation (14). Following the usual procedure of combining the denominators, shifting the integration variable and then carrying out the momentum integration, one obtains for the LHS of equation (14)

$$\text{LHS of equation (14)} = 2\mathrm{i}\pi^2 \not{p} gf + \frac{\mathrm{i}\pi^2}{\lambda} \int_0^1 \mathrm{d}x \frac{(1-x)^\lambda}{(L^2)^\lambda} [\mathrm{i}m_\psi p^2(1-x)$$
$$+ 2gf\not{p}x - \not{p}m_\psi^2] + \frac{\mathrm{i}\pi^2}{\lambda} \int_0^1 \mathrm{d}x \frac{(1-x)^\lambda}{(L'^2)^\lambda} [\mathrm{i}m_\psi p^2(1-x) + 2gf\not{p}x + \not{p}m_\psi^2]. \tag{16}$$

In equation (16) $L^2$ and $L'^2$ are given by

$$L^2 = -p^2x^2 - (2gf - p^2)x + m_\psi^2 \tag{17a}$$

$$L'^2 = -p^2x^2 + (2gf + p^2)x + m_\psi^2. \tag{17b}$$

To carry out the $x$ integration in equation (16) we expand the integrand into powers of $\lambda$ and retain terms up to order $\lambda$. With this, we have

$$\text{LHS of equation (14)} = -\mathrm{i}\pi^2 \not{p} gf + \tfrac{1}{2}\pi^2 m_\psi p^2$$
$$+ \frac{\mathrm{i}\pi^2}{\lambda}(\mathrm{i}m_\psi p^2 + 2gf\not{p}) + \pi^2 m_\psi^2 p^2 \int_0^1 \mathrm{d}x (1-x)(\ln L^2 + \ln L'^2)$$
$$- 2\mathrm{i}\pi^2 gf\not{p} \int_0^1 \mathrm{d}x\, x(\ln L^2 + \ln L'^2)$$
$$+ \mathrm{i}\pi^2 \not{p} m_\psi^2 \int_0^1 \mathrm{d}x (\ln L^2 - \ln L'^2). \tag{18}$$

Following exactly the same procedure the RHS of equation (14) is evaluated. The result is

$$\text{RHS of equation (14)} = -\frac{8\pi^4}{g}(c - m_\psi f) + (2gf + \mathrm{i} m_\psi \not{p}) \times \left( -\mathrm{i}\pi^2 \frac{\not{p}}{2} + \mathrm{i}\pi^2 \frac{\not{p}}{\lambda} - \mathrm{i}\pi^2 \not{p} \int_0^1 \mathrm{d}x\, x(\ln L^2 + \ln L'^2) + \pi^2 m_\psi \int_0^1 \mathrm{d}x(\ln L^2 - \ln L'^2) \right). \tag{19}$$

Now substituting expressions (18) and (19) into equation (14) and making cancellations from both sides we obtain

$$p^2 \int_0^1 \mathrm{d}x(1-2x)(\ln L^2 + \ln L'^2) - 2gf \int_0^1 \mathrm{d}x(\ln L^2 - \ln L'^2) = \frac{8\pi^2}{m_\psi g}(m_\psi f - c). \tag{20}$$

Next evaluating the consistency condition in equation (12*a*) explicitly we have

$$(8\pi^2/m_\psi g)(m_\psi f - c) = (m_A^2 \ln m_A^2 - 2m_\psi^2 \ln m_\psi^2 + m_B^2 \ln m_B^2). \tag{21}$$

With $L^2$ and $L'^2$ given by equations (17) we perform the $x$ integration on the LHS of equation (20). The $x$ integration is lengthy but simple and it is easy to check that the final result is equal to the RHS of equation (21), thus verifying the Ward identity of equation (9).

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